# Investment Under Adverse Selection and Low Interest Rates

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**Abstract**

In the aftermath of the recent financial crisis, central banks have responded by setting the interest rates to unprecedentedly low levels. Yet, the impact of such low rates on investment and growth has been weaker than expected. I argue in this paper that low interest rates may unfavourably impact investment because they exacerbate the adverse selection problem in financial markets. Under low interest rates, low-quality entrepreneurs search for high-return investment opportunities. To prevent a worsening of the pool of borrowers, financial intermediaries react by raising their lending standards. Therefore, even high-quality entrepreneurs with insufficient own-funds are denied financing and the economy is characterised by low aggregate investment and growth. This phenomenon is perhaps more detrimental in the aftermath of crises, where the average quality of risky projects and the average entrepreneurial wealth have deteriorated.

KEYWORDS: Interest rates, borrowing, investment, entrepreneurial wealth, adverse selection

JEL CLASSIFICATION: D82, E30, E44, E58, G01, G21

## 1 INTRODUCTION

The recent financial crisis was triggered by subprime mortgage delinquencies and foreclosures but quickly spread over the whole financial system and inevitably the real economy. As a consequence, most nations experienced their most severe downturn since the Great Depression. The US economy contracted by almost 3% by the end of 2009 and only recently showed signs of revival. To avoid a complete meltdown, central banks worldwide

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responded by setting the interest rates to unprecedentedly low levels. Never before in recent history have real interest rates in advanced economies fallen to near zero (e.g., US) or become negative (e.g., Euro Area, Japan, etc.). Nonetheless, the effect of such low rates on investment and growth has been weaker than expected. The annual rate of output growth in the US and elsewhere is still below 2%. This very fact constitutes a puzzle for academic economists and policy makers alike. A quote from the Economist best describes this puzzle:[1] *"IT'S ONE of the fundamental lessons of any introductory economics course: lower interest rates, when all else remains equal, leads to higher levels of investment. But today, after several years of near-zero interest rates and only modest increases in investment to show for it, some economists are claiming just the opposite..."*. Why such unprecedentedly low interest rates have not boosted investment as expected?

In this paper, I provide a possible answer to this question. I argue that low interest rates might unfavourably impact investment because they exacerbate the adverse selection problem in financial markets. Under low interest rates, low-quality entrepreneurs search for risky, high-return projects. This restricts the access to credit to high-quality entrepreneurs with insufficient own-funds. To flesh out this argument, I build a simple model with financially constrained entrepreneurs who seek to finance risky projects from financial intermediaries. There are two possible entrepreneur types, who differ in the riskiness of their projects. High-type entrepreneurs have a higher probability to succeed than low-type ones. Financial intermediaries are unable to identify the true type of an entrepreneur leading to a natural "lemons" problem. Adverse selection is pernicious enough to lead to a complete market shutdown if entrepreneurs are totally cashless. This gives scope to entrepreneurial wealth to partially alleviate the adverse selection problem. I assume that entrepreneurs are wealth heterogenous.

The safe interest rate, controlled by the central bank, is the interest rate at which financial intermediaries can raise the necessary funds from depositors to grant loans, as well as the rate at which entrepreneurs can save any excess funds. Therefore, a trade-off arises for an entrepreneur: investing in the safe rate or the risky project. This trade-off is the driving mechanism of the paper. Even though, any high-type entrepreneur strictly prefers to invest in the risky project over the safe rate, under a low loan-repayment rate, so does a low-type. Rationally expecting this, financial intermediaries raise the lending standards. Only entrepreneurs with sufficient wealth are able to borrow. A high-enough share in the project is too costly for a low-type entrepreneur who would rather invest in the safe rate, as her project has low probability to return positive output. As such, a low safe rate exacerbates the adverse selection problem, i.e., it makes the risky project more attractive to low-type entrepreneurs. This constrains high-type entrepreneurs who are now demanded a higher share in the risky project. Therefore, high types with insufficient own-funds are denied financing and forced to invest in the safe interest rate. This leads

[1]Free Exchange, The Economist, November 12th 2015.

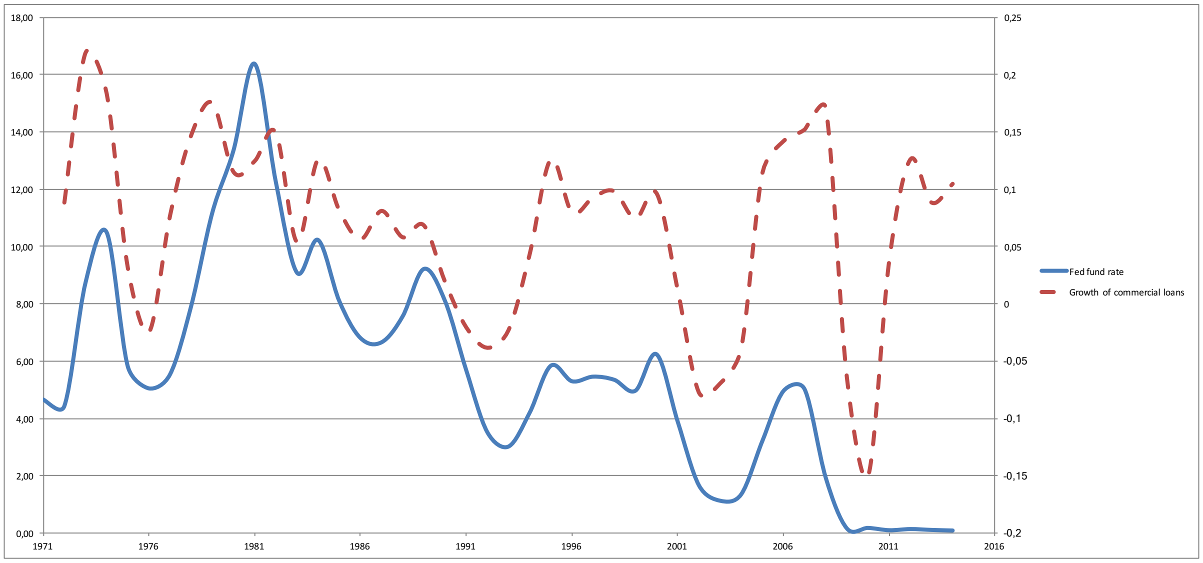


Figure 1: The Fed Funds rate and the growth of Commercial and Industrial Loans granted by US financial institutions. The left vertical axis depicts the Fed fund rate and the right one the growth of Commercial and Industrial Loans.
**Source:** Federal Reserve Bank

to insufficient investment and consequently weak growth.

The model predicts that the effect of low interest rates on investment is more detrimental during, or in the aftermath of, recessions. For, recessions usually shift the cumulative distribution of entrepreneurial wealth upwards. This means that a smaller share of entrepreneurs has access to credit and, hence, investment stagnates. The main lesson of the analysis is that responding to recessions by setting interest rates extremely low might in fact aggravate the downturn rather than boosting investment. Instead, a central bank should keep interest rates at a reasonable level before the market shows signs of revival.

Figure 1 plots the Federal funds rate along with the growth rate of commercial loans granted from US banks for the period 1971-2015. With the only exception the period 1980-1982, these two seem to follow a surprisingly similar pattern. Low rates are accompanied by low growth, whilst high rates by high growth. In the aftermath of the crisis, the Federal Funds rate has been approximately 0.5%, whilst the growth rate of commercial loans has been approximately 10%, as high as it was during the second half of the 80's and 90's when the Fed funds rate was approximately 6.5% and 5.5% respectively.

The literature of credit markets with asymmetric information is very rich. Perhaps the most closely related papers are the seminal papers by Stiglitz and Weiss (1981) (hereafter, SW) and De Meza and Webb (1987) (hereafter, DW). In both papers, the equilibrium is characterised by pooling of types at the same interest rate. In particular, SW show that debt financing can lead to credit rationing in equilibrium if banks have no sorting device other than the interest rate. DW show that by slightly changing the fundamentals in the SW model, the opposite result prevails; there is excessive investment in equilibrium. My model differs from these of SW and DW in at least two respects. First, I consider a model with two possible risk types as opposed to the model of SW and DW who consider continuum of types. Second, and perhaps more importantly, in my model types differ with respect to their entrepreneurial wealth as opposed to SW and DW whose models feature entrepreneurs with the same amount of initial wealth. As I show, this substantial element introduces a trade-off between investment in the safe and the risky asset which provides fruitful insights.

The paper is also related to the long-standing literature examining the role of net worth and/or collateral in mitigating credit market imperfections. For instance, Bester (1985, 1987) highlights the role of collateral as a possible sorting device in the SW model. Collateral has a higher implicit cost for low-type entrepreneurs who get to default more often. An implication of this strand of the literature is that abundance of collateral causes investment to boom. In my model, there is no collateral as entrepreneurial wealth is highly liquid and hence non-pleadgable. However, entrepreneurial wealth plays an important role in alleviating the adverse selection problem similar to this played by collateral in Bester (1985, 1987). Myers and Majluf (1984) highlight the potential equity underpricing of new financing due to adverse selection. The result forms the ground for the celebrated "pecking-order hypothesis", according to which companies prefer internal financing (e.g., retained earnings) to debt to equity. Holmstrom and Tirole (1997) emphasise the role of entrepreneurial wealth to mitigate agency problems. Entrepreneurs select to stay or not to stay diligent after receiving financing. As expected, diligence comes at cost. With no initial wealth, there is no financing because incentives to stay diligent are weak. Entrepreneurial wealth increases the cost of shirking and, hence, for a sufficiently high share in the project, an entrepreneur is able to guarantee financing. Even though there are similarities between my model in this studied by Holmstrom and Tirole (1997), entrepreneurs face a different trade-off between investment in the interest rate and the risky project. Hence, our predictions differ significantly.

Lastly, the role of entrepreneurial wealth (i.e., net worth) in explaining business cycles holds a prevalent position in the macro-finance literature since the seminal contribution of Bernanke and Gertler (1989). These authors show that small shocks are amplified through a "financial accelerator". The mechanism that drives the result relies on the wedge between the implicit cost of internal and external financing. Because of this wedge, firms rely on internal funds to finance positive net present value projects. Since building one's net worth requires time, an unanticipated shock that reduces net worth tends to persist. However, the underlying capital imperfection is different between this paper and Bernanke and Gertler (1989). In Bernanke and Gertler (1989) lenders can enforce repayments only through costly monitoring as in Townsend (1979), whilst in my model asymmetric information is at the interim stage.

The remainder of the paper is organised as follows. In Section 2 I describe the model.

In Section 3, I solve the model and I do comparative statics. In Section 5, I conclude the paper.

## 2 The Model

■ **Entrepreneurs and Projects.** There is a continuum of entrepreneurs. An entrepreneur is characterised by a type $i \in \{H, L\}$ and a wealth level $w \in [0, \bar{w}]$. Only the entrepreneur knows her true type. For simplicity, suppose that $i$ and $w$ are independently distributed with $\lambda_i$ denoting the probability $i$ to be realised and $F(w)$ the probability function (cdf) of $w$, which is assumed to be continuous with full support. Each entrepreneur has a project. By investing $I > \bar{w}$ dollars, an entrepreneur of type-i can realise payoff $X$ with probability $\pi_i$ or zero otherwise. To reduce the number of relevant parameters, let $\pi_H = \pi$ and $\pi_L = \gamma\pi$, where $\gamma < 1$. For future notational convenience, let $\bar{\gamma} = \lambda_H + \lambda_L \gamma$. Even though, the project of type H has a higher probability to succeed, it also has a higher cost. For simplicity, I assume that the cost of the project of type H is equal to $K$, whilst this of a type L is, for simplicity, equal to zero. This cost can be given a dual interpretation. It can either be considered as an outside option, or as the cost of effort to set up the project. In either case, it is natural to assume that the cost for type-H is strictly higher than this for type L. The real (gross) risk-free interest rate (or simply interest rate) is given by $r > 1$. I assume that this interest rate is controlled by the central bank.

□ **Assumptions.** I restrict attention to environments that satisfy the following set of assumptions:

**Assumption 2.1.** $\bar{\gamma}\pi X - Ir < 0$

This assumption has two main implications. First, because $\bar{\gamma} > \gamma$, it implies that type L has a negative net present value (NPV) project. Second, as it will become clear shortly, it eliminates the, rather uninteresting, case in which in equilibrium every entrepreneur, regardless of her wealth, receives financing and invests in the risky project. In fact, Assumption 2.1 gives scope to entrepreneurial wealth to prevent a collapse of the financial market due to adverse selection.

**Assumption 2.2.** $K < (\frac{1}{\gamma} - 1)\bar{w}r$

Assumption 2.2 also has a dual implication. First, it implies that at least the wealthiest type-H entrepreneurs are able to borrow and invest in the risky project. Second, combined with Assumption 2.3 below, it implies that type H has a positive NPV project.

**Assumption 2.3.** $\pi X - Ir - (\frac{1}{\gamma} - 1)\bar{w}r > 0$

Assumption 2.3 rules out the case in which type-H entrepreneurs, regardless of their wealth, can borrow at an interest rate that reflects their true probability of default, without type L desiring to imitate them.

I make two assumptions on the space of feasible loan contracts,. First, entrepreneurs are protected by limited liability. Second, the initial wealth can be invested in the project but cannot be pledged for future repayments to the bank. The second assumption implies that the initial wealth is in the form of cash and hence non-pledgeable. Due to these two assumptions and the binary nature of the projects, i.e., success/failure, there is no loss of generality in concentrating on simple, risky debt contracts. A risky debt contract is denoted by $(x, R) \in \mathbb{R}^2_+$, where $x$ is the amount of wealth the entrepreneur is asked to invest and $R$ is the share from the return of the project that is pledged as a repayment for the amount borrowed. Because of limited liability and the assumption of non pledgeability of the initial wealth, $R$ is paid only in case the project realises strictly positive output. I assume that entrepreneurs do not renege on their repayment promises, abstracting that way from commitment or imperfect enforcement issues. This allows me to concentrate on the effect of adverse selection on aggregate investment.

I assume that entrepreneurial wealth is observable, even though, as it becomes clear, this assumption plays no role in the qualitative features of the results. Its main purpose is to simplify the notation and the analysis as it permits the loan contracts to be made contingent on the observable wealth of an entrepreneur.[2]

□ **Timing of Events and Equilibrium.** I now describe the timing of events. There are two periods. All agents consume solely in the second period. The interest rate from the first to the second period is equal to $r$, as specified previously. In period one, financial intermediaries enter the market and offer loan contracts. Financial intermediaries are risk-neutral and can raise as much capital as they wish at the risk-free interest rate. Each financial intermediary can offer a menu of loan contracts of the following form: $\big(x(w), R(w)\big)$. Therefore, a loan contract specifies for an entrepreneur with wealth $w$ the amount of loan and the repayment rate. In the second stage, each entrepreneur selects at most one contract from from one financial intermediary. Alternatively, an entrepreneur can invest her wealth in the risk-free interest rate. In the last stage, those entrepreneurs who borrowed from a financial intermediary, invest in the risky project, whereas those who did not, invest in the risk-free interest rate. In the second period, the return of the project is realised (success or failure) and all agents consume. Because agents consume only in the second period, those entrepreneurs who do not invest in the project in period one will invest the entirety of their wealth in the risk-free interest rate.

An equilibrium is a set of contracts such that, when entrepreneurs choose contracts: (i) no contract in the equilibrium set makes negative expected profits; and (ii) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit. This

[2]Suppose instead that wealth was unobservable but that the entrepreneur could voluntarily reveal to the bank any part of it. Then the loan contracts should be made contingent on the revealed part of the wealth. That would require one extra variable to be specified, i.e., the part of the wealth that the entrepreneur decided to reveal to the bank. The results remain the same under either modelling approach.

definition is similar to the one adopted by Rothschild and Stiglitz (1976) in their study of the competitive insurance market with adverse selection. Nonetheless, as I show in the next section, unlike Rothschild and Stiglitz (1976), in the model I study an equilibrium always exists and entails pooling of types.[3]

## 3 ANALYSIS

■ **Equilibrium Contracts.** In this section, I characterise the set of equilibrium contracts in a series of lemmas and propositions.

To begin with, as I show in the following lemma, one can ameliorate the notational burden by restricting the set of relevant menus of loan contracts to not specify the type of the entrepreneur.

**Lemma 3.1.** *For every $w \in [0, \bar{w}]$, there exists no equilibrium in which $(x_H(w), R_H(w)) \neq (x_L(w), R_L(w))$.*

*Proof.* Suppose to the contrary that there exists an equilibrium in which $(x_H(w), R_H(w)) \neq (x_L(w), R_L(w))$. Because of free-entry on the supply side, $R_i(w) = \frac{(I - x_i(w))r}{\pi_i}$ for every $i$. The payoff of type L from the contract of type H, with the same initial wealth $w$, is given by:

$$\gamma\pi\Big(X - \frac{(I - x_H(w))r}{\pi}) + (w - x_H(w)\Big)r = \gamma(\pi X - Ir) - (1-\gamma)x_H(w)r + wr \quad (3.1)$$

whereas this from contract $(x_L(w), R_L(w))$:

$$\gamma\pi X - Ir + wr \quad (3.2)$$

In equilibrium the payoff given in Eq. (3.2) needs to be weakly greater than this given in Eq. (3.1), or:

$$\gamma\pi X - Ir + wr \geq \gamma(\pi X - Ir) - (1-\gamma)x_H(w)r + wr$$

This means that $x_H(w) \geq I$, which clearly contradicts that $I > \bar{w}$. □

In words, Lemma 3.1 states that no equilibrium exists in which the two types with the same amount of initial wealth select different contracts. Thanks to Lemma 3.1, we can solely focus on the characterisation of equilibrium menus of contracts of the form $(x(w), R(w))$. First, I specify when, if at all, type i with initial wealth $w$ borrows and invests in the risky project in equilibrium.

[3]Recall that in Rothschild and Stiglitz (1976), (i) an equilibrium, when it exists, entails separation of types, and, (ii) an equilibrium might not exist when the share of low-risk types in the population is high enough.

**Lemma 3.2.** *For every $w \in [0, \bar{w}]$, there exists no equilibrium in which type L borrows and invests in the risky project.*

*Proof.* Because of Assumption 2.1, an equilibrium in which type L with wealth $w$ invests, whilst type H with wealth $w$ does not, is not sustained. Suppose that there exists an equilibrium in which type L with wealth $w$, along with type H, borrows and invests in the risky project. Due to Lemma 3.1, only pooling equilibria in which both types with the same initial wealth borrow the same amount and pay the same interest rate potentially exist. Hence, let the equilibrium contract be $(\bar{x}(w), \bar{R}(w))$. Because of free-entry competition on the supply side, $\bar{R}(w) = \frac{(I-\bar{x}(w))r}{\bar{\gamma}\pi}$. Consider the payoff of type L from contract $(\bar{x}(w), \bar{R}(w))$:

$$\gamma\pi(X - \bar{R}(w)) + (w - \bar{x}(w))r = \gamma(\pi X - \frac{Ir}{\bar{\gamma}\pi}) + (\frac{\gamma}{\bar{\gamma}} - 1)\bar{x}(w)r + wr \tag{3.3}$$

Because the payoff of type L with wealth $w$ by investing in the risk-free interest rate is $wr$, the payoff given in Eq. (3.3) has to be weakly greater than $wr$. This means that either $\gamma > \bar{\gamma}$, or $\pi X - \frac{Ir}{\bar{\gamma}\pi} > 0$ or both. However, we know, by definition, that $\gamma < \bar{\gamma}$ and, from Assumption 2.1, that $\pi X - \frac{Ir}{\bar{\gamma}\pi} < 0$. Hence, we have a contradiction. □

This result relies on Assumption 2.1. The intuition is as follows. If both types with initial wealth $w$ invest in the risky project, thanks to Lemma 3.1, the only possibility is that the repayment rate to the bank reflects the weighted average probability of the two types to succeed. one can show that, under Assumption 2.1, this repayment rate is too high for a type-L entrepreneur to invest for every possible amount of initial wealth. Therefore, such an equilibrium cannot be sustained.

The next step is to examine under what conditions an equilibrium in which at least type H invests in the risky project exists. I show in the following proposition that such an equilibrium indeed exists.

**Proposition 3.3.** *In equilibrium, type H invests in the risky project if and only if $w \geq \frac{K}{(\frac{1}{\gamma}-1)r}$.*

*Proof.* I first prove the "only if" part. Suppose to the contrary that type H with wealth $w < \frac{K}{(\frac{1}{\gamma}-1)r}$ invests in the risky project. Because of Lemma 3.2, the only possibility is that only type H borrows and invests in the risky project. Suppose that the equilibrium contract is $(\bar{x}(w), \bar{R}(w))$. Because of free-entry competition on the supply side, there are two possibilities. Either $\bar{R}(w) = \frac{(I-\bar{x}(w))r}{\pi}$, or $\bar{R}(w) = X - \frac{\bar{x}(w)r}{\gamma\pi}$. In the first case, I show that type L with wealth $w < \frac{K}{(\frac{1}{\gamma}-1)r}$ prefers to invest in the risky project. Suppose not. The payoff of type L from such contract is:

$$\gamma\pi(X - \bar{R}(w)) + (w - \bar{x}(w))r = \gamma(\pi X - Ir) - (1-\gamma)\bar{x}(w)r + wr$$

This needs to be weakly less than $wr$. Therefore, the following is true:

$$\pi X - Ir - (\frac{1}{\gamma} - 1)\bar{x}(w)r \leq 0$$

Because $\bar{x}(w) \leq w < \bar{w}$, this clearly contradicts Assumption 2.3. Now suppose that $\bar{R}(w) = X - \frac{\bar{x}(w)r}{\gamma\pi}$. At this interest rate, type L with wealth $w < \frac{K}{(\frac{1}{\gamma}-1)r}$ is indifferent between investing in the risky project and the safe interest rate. The payoff of type H is:

$$\pi(X - X + \frac{\bar{x}(w)r}{\gamma\pi}) + (w - \bar{x}(w))r - K = (\frac{1}{\gamma} - 1)\bar{x}(w)r + wr - K$$

If this payoff is weakly greater than $wr$, then:

$$\bar{x}(w) \geq \frac{K}{(\frac{1}{\gamma} - 1)r},$$

which means that $\bar{x}(w) > w$. This clearly contradicts that $\bar{x}(w) \leq w$.

To prove the "if" part, suppose that in equilibrium, type H with wealth $w > \frac{K}{(\frac{1}{\gamma}-1)r}$ does not invest in the risky project. Consider contract $(\tilde{x}(w), \tilde{R}(w))$, where $\tilde{R}(w) = X - \frac{\tilde{x}(w)r}{\gamma\pi} + \epsilon$. The payoff of type L from this contract is:

$$\gamma\pi(X - X + \frac{\tilde{x}(w)r}{\gamma\pi} - \epsilon) + (w - \tilde{x}(w))r = wr - \gamma\pi\epsilon < wr$$

Therefore, type L strictly prefers to invest in the risk-free interest rate. The payoff of type H is:

$$\pi(X - X + \frac{\tilde{x}(w)r}{\gamma\pi} - \epsilon) + (w - \tilde{x}(w))r - K = (\frac{1}{\gamma} - 1)\tilde{x}(w)r + wr - \pi\epsilon - K$$

For $\tilde{x}(w) = w$ and $\epsilon > 0$ small enough it is true that:

$$(\frac{1}{\gamma} - 1)wr - \pi\epsilon - K > 0$$

which means that type H strictly prefers contract $(w, \tilde{R}(w))$ to investing in the risk-free interest rate. The profit of this contract, if taken only by type H, is:

$$\pi\tilde{R}(w) - (I - w)r = \pi(X - \frac{wr}{\gamma\pi} + \epsilon) - (I - w)r = \pi X - Ir - (\frac{1}{\gamma} - 1)wr + \pi\epsilon > 0$$

where the last inequality follows from Assumption (2.3) and the fact that $w < \bar{w}$. Therefore, there is a contradiction with requirement (ii) of the definition of the equilibrium. $\square$

Proposition 3.3 plays a key role in the analysis of the paper. It states that type-H entrepreneurs with sufficiently low wealth are not able to guarantee financing. The intuition behind this result is that a type-H entrepreneur with sufficiently low wealth needs to pay a sufficiently low interest rate to a financial intermediary, i.e., an interest rate that reflects her true probability to succeed, to has a payoff greater than the one she can guarantee by investing in the safe interest rate. At such low interest rate however, type L with the same wealth finds it profitable to invest in the risky project and hence the financial intermediary providing this loan necessarily makes losses. Hence, a financial intermediary prefers to decline financing to all entrepreneurs with sufficiently low wealth.

**Proposition 3.4.** *The unique equilibrium payoffs of a type-L and a type-H entrepreneur with initial wealth $w$ are $wr$ and $u_H(w)$ respectively, where:*

$$u_H(w) = \begin{cases} wr, & \text{if } w \leq \frac{K}{(\frac{1}{\gamma}-1)r} \\ \frac{1}{\gamma}wr - K, & \text{if } \frac{K}{(\frac{1}{\gamma}-1)r} \leq w < \frac{\pi X - Ir}{(\frac{1}{\gamma}-1)r} \\ \pi X - Ir - K, & \text{if } \frac{\pi X - Ir}{(\frac{1}{\gamma}-1)r} \leq w \leq \bar{w} \end{cases} \tag{3.4}$$

*Proof.* It was shown in Lemma 3.2 that type L, regardless of her wealth, invests in the risk-free interest rate. Moreover, it was shown in Proposition 3.3 that if $w < \frac{K}{(\frac{1}{\gamma}-1)r}$, type H invests in the the risk-free interest rate, whereas if $w \geq \frac{K}{(\frac{1}{\gamma}-1)r}$, type H invests in the risky project. Let the equilibrium contract be denoted by $(\bar{x}(w), \bar{R}(w))$. Because of free-entry competition on the supply side, the two possible repayment rates are $\frac{(I-\bar{x}(w))r}{\pi}$ and $X - \frac{\bar{x}(w)r}{\gamma\pi}$. Due to Assumption 2.3, the latter is always higher than the former. To characterise the wealth level above which the second repayment rate is in force, let us write the payoff of type L if she invests in the risky project at that repayment rate:

$$\gamma\pi(X - \frac{(I-\bar{x}(w))r}{\pi}) + (w - \bar{x}(w))r = \gamma\pi X - \gamma Ir - (1-\gamma)\bar{x}(w)r + wr$$

For a type-L entrepreneur with wealth $w$ not to want to invest in the risky project, it has to be that:

$$\gamma\pi X - \gamma Ir - (1-\gamma)\bar{x}(w)r + wr \leq wr$$

or

$$\bar{x}(w) \geq \frac{\pi X - Ir}{(\frac{1}{\gamma}-1)r}$$

Therefore, the lowest possible wealth for which the repayment rate is the one that reflects the true probability of type H to succeed is $\frac{\pi X - Ir}{(\frac{1}{\gamma}-1)r}$. $(\bar{x}(w), \bar{R}(w))$ needs to satisfy the following :

$$
\bar{R}(w) = \begin{cases} X, & \text{if } \; w < \frac{K}{(\frac{1}{\gamma}-1)r} \\ \\ X - \frac{\bar{x}(w)r}{\gamma\pi}, & \text{if } \; \frac{K}{(\frac{1}{\gamma}-1)r} \leq w < \frac{\pi X - Ir}{(\frac{1}{\gamma}-1)r} \\ \\ \frac{(I-\bar{x}(w))r}{\pi}, & \text{if } \; \frac{\pi X - Ir}{(\frac{1}{\gamma}-1)r} \leq w \leq \bar{w} \end{cases}
$$

The payoffs for type H are unique and given in Eq. (3.4). □

□ **Comparative Statics.** The key equation is the one defining the cutoff wealth below which investment in the risky project by type H is not possible. This cutoff is defined as:

$$
\underline{w}(r) = \frac{K}{(\frac{1}{\gamma} - 1)r} \tag{3.5}
$$

which is clearly a decreasing function of $r$. In other words, one obtains the following corollary:

**Corollary 3.5.** *An increase in the risk-free interest rate can cause an increase in investment in risky projects by type H.*

The aggregate investment in the risky project is defined by the share of type-H entrepreneurs who borrow and invest. Using Eq. (3.5), one can characterise this share as follows:

$$
\iota(r) = \lambda_H\big(1 - F(\underline{w}(r))\big) \tag{3.6}
$$

which, evidently, is higher the higher is $\lambda_H$, for every interest rate. Because $\underline{w}(r)$ is strictly decreasing in $r$, the effect of a low interest rate on investment further depends on the shape of $F(w)$. The following result requires no formal proof:

**Proposition 3.6.** *For every interest rate, investment falls if the cdf of wealth shifts upwards.*

Proposition 3.6 implies that the impact of low interest rates is more dramatic when the average entrepreneurial wealth has deteriorated. This is more likely true during recessions. For instance, after a financial crisis, the cdf of wealth might shift upwards, perhaps in a *First Order Stochastic Dominance* sense. When the cdf of wealth shifts upwards, for any level of wealth strictly less than $\bar{w}$, there is a larger share of entrepreneurs possessing that wealth. Therefore, there is less aggregate investment.

## 4 Conclusion

In this paper, I studied a simple model of a credit market with adverse selection and wealth-heterogeneous entrepreneurs. Adverse selection was severe enough to cause a collapse of the market if entrepreneurs were totally cashless. I showed that entrepreneurs faced a trade-off between investing in the risk-free interest rate and a risky project whose quality was private information. In equilibrium, high-type entrepreneurs with adequate wealth could borrow and invest in the risky project, whilst low-type entrepreneurs as well as high-type entrepreneurs with insufficient wealth invested in the risk-free interest rate. The model predicted that low risk-free interest rates might be detrimental to investment because they exacerbated the adverse selection problem. Banks raised the lending standards to discourage low-type entrepreneurs from borrowing. This result might be more severe during crises, where the distribution of entrepreneurial wealth has shifted upwards. Implications of the model were discussed regarding the effect of near-zero interest rates to investment.

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